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# Ensuring Control Accuracy

With 80 Figures



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## **Ensuring control accuracy**

The methods of analysis and synthesis of nonadaptive automatic control systems and other dynamic systems, capable of providing an acceptable or the highest guaranteed level of operation accuracy under the allowable uncertainty of excitation properties and system characteristics are stated. Control accuracy is estimated by the upper bound of the total r.-m.-s. or maximal error. When giving the class of input excitations, the restriction of possible values of derivatives, generalized moments of spectral densities, width of a spectrum and other numerical characteristics, which are easily controllable in practice are used. The research on frequency and time domains are based on mathematical results of the general problem of moments, the theory of disturbance accumulation and numerical methods.

This book is aimed towards specialists in designing and researching control systems, engineers, postgraduate, and students.

### Preface

Dynamic systems, described by definite differential or difference equations, are the universal model for investigation of laws in behavior of different natural processes related to the motion of material objects, information transfer, and the development of social, biological, economic and other structures. Their application in the investigation of control processes in technical and other systems forms the fundamental idea of automatic control theory.

Estimations of dynamic system accuracy greatly depend on the choice of initial models of actions. Those models give input for actuating the right member of appropriate differential equations. Variability and partial knowledge of real action properties trouble the formation of spectral-correlation and other full models of action, described by the analytical functions of nontrivial mode. This is due to the loss of practical efficiency of investigation (due to low validity of such models) which exceeds the possible gain of more accurate system adjustment on the definite operating regime. Therefore it is rational to use for description of action properties the nonparametric classes of functions. Their satisfactory width provides the required validity of description. This approach requires the investigation and development of special methods for analysis and synthesis of dynamic systems with the ensured accuracy characteristics.

Ensuring good protection of results from errors in the initial data is important not only in researching control accuracy. A similar requirement applicable to mathematical statistics problems was clearly formulated by P. Huber [61]. It was indicated as a "robustness" term in the sense of its insensitivity to low deviation from initial suppositions. The robustness in the modern theory of automatic control is often coupled to the ensuring of system stability at the definite scatter of its parameters (V. L. Kharitonov theorem, discovered by J. Z. Tsypkin and his school). Therefore the more widespread explanation of its concept became convenient. If the system or algorithm posses the high efficiency at the nominal operation conditions and good efficiency at the deviation from the nominal conditions in the preset accessible limits [3, 8, 27, 40, 54] then it is considered robust.

These limits can be determined by the accepted classes of external and parametric disturbances. Ensuring control accuracy in this case can be treated as providing the robust accuracy, and then appropriate control systems are called robust.

The robustness concept is actually not new. It is followed by the tendency to give nonadaptive systems the property of holding the preset characteristics in the admissible limits at possible variation of their operating conditions, without demanding the best quality for some fixed conditions. Highly experienced designers have always been working in that way. Their works were based not only upon any mathematical theories but also upon the prudence and good intuition in major cases. The theoretical methods of dynamic systems investigation in their development could not comprise all the features of practical design problems. Some "residual" in theoretical and practical approaches to design process always existed. It stimulated the improvement of theories and created witty stories about the loss of mutual understanding between theorists and empirics. The limited possibilities of theoretical investigations are coupled to excessive formalization and idealization of problem statements. It can also be the fee for the possibility of finding the strict solution. The robust approach is the attempt to smooth the sharpness of exposed problems in the account of the more rough, approximate description of initial information about the conditions of system operation, assuming the possibility of normality in such conditions.

This book does not comprise all known methods for ensuring control accuracy, which are various enough and can hardly be joined with anything to form a single theory, except with the robust concept used in all of them.

The chosen stated methods suppose mostly the investigation in the frequency domain. The main arguments in advantage of such choice are their relative simplicity and high validity of initial conditions being appreciated by all specialists and practitioners. The direct interaction between those methods and classical frequency domain methods for investigation of automatic control systems quality is also very important. It simplifies the understanding of material for a large group of readers, including students. Therefore, a list of questions is given at the end of every chapter. The statement style can satisfy system designers and designers of automatic control devices. The author hopes that specialists in the sphere of control theory will read this book, although many of them suppose the frequency domain to be some kind of trampled out field of knowledge, where it is hard to find something new. But the author was not seeking the new but required in practice, investigation methods, which are improved by the given formulae and examples.

The main list of references contains minimum basic publications that are close to the subject of the book. Additional literature is indicated by particular references in the text.

Most of the ideas shown in this book were formulated by the authors' cooperation with his scientific teacher V. A. Besekerski. He became the co-author when publishing the first book devoted to robust control systems [8] based on Besekerski fundamental treatises [6, 7]. The author gratefully appreciates Prof. Besekerski for an excellent education and support.

Professor A. A. Zinger was the first to confirm the authors' suppositions about the possibility of investigating the accuracy of linear filtration of signals with the limited variances of derivatives on the basis of using the current problem apparatus by Chebyshev-Markov. He also recommended the perfect treatise by M.G. Krein and A.A. Noodelman. By virtue of those treatise the author has discovered the new vision on familiar verity.

Professor I.B. Chelpanov with his opinions on earlier authors' publications, and with his speeches and books during many years has helped the author to keep certainty in the efficiency of chosen investigation methods.

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Academician F.L. Chernousko has greatly supported the author at the most difficult period of the book's completion, which predetermined the possibility of publishing this book.

The>author>had>the>additional>opportunity>to>check>up>the>efficiency>of>suggested>methods>for>theoretical investigations>in>contensive>problems>of>synthesizing>precision>automatic>control>complex>for>the>relative>motion>of>aerospace planes>at>their>horizontal>start>and>anding>on>moving>kranoplanes>[86,>89].>The concept>for>construction>of>this>perspective space transport system is being developed>in>cooperation>with>professor>N.>Tomita>and>other>colleagues>from>Tokyo. The>communication>between>them>provided>new>ideas>for>improving>the>material in the book.

On>every>tages>of>this>book>preparation>the>author>was>inspired>by>support>of many>other>friends>and>colleagues,>who>are>greatly>faithful>to>science>and>who give all the strength they have to it. Many thanks to all of them.

St. Petersburg, December 2003

>>>>Alexander Nebylov

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## Chapter 1

## Main concepts and definitions

#### §1.1 Dynamic systems

The dynamic system concept. One of the major results of the theoretical cognitive process is the possibility to evaluate the consequences of different human or natural force actions on those or other complicated objects or systems, capable to accept such actions. It is impossible to discover correct solutions when constructing engineering devices, technological processes, socioeconomic mechanisms, and elements of human environment or when excluding it in other areas of intellectual activity.

Real systems of theoretical research are represented, such as models having a certain formal description, which are more often mathematical. The legitimacy of model choice is proved by the adequacy of real systems and model responses to excitations of an identical kind.

The mathematical model of a system enables one to find the relationship between some input g (setting action) and output y (response). The external actions and responses can be described thus by functions of continuous time t or discrete time n and look like y(t) = y(t,g) or y(nT) = y(nT,g), where g = g(t) or g = g(nT); T is the discretization period, n = 0, 1, 2, ... (Fig 1.1).



Fig. 1.1

It is convenient to use for research, for example, the unit step-function g(t) = 1(t) or g(nT) = 1(nT) as a trial action. The system reaction to such a step generally is not a step and is described by a smoother function. The form of such function y(t) or y(nT) characterizes the dynamic properties of a system. The system possessing such dynamic properties is called the dynamic system.

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Thus, input and output actions in dynamic system are functions of time, and the current value of output is determined not only by current, but also by previous values of input, i.e. the system has some "memory" persistence. The mathematical model of dynamic systems is inhomogeneously differential or a difference equation, whose left member is written concerning a response, and the right member includes the external action.

One of the possible definitions for a dynamic system is: *the structure, into* which something (substance, energy or information) is entered in the defined moments and from which outputs something in any instant. It is used to describe a cause-effect correlation from the past into the future.

**Examples of dynamic systems.** Give examples concerning the area of engineering systems (1 - 4), physics-biological systems (5 - 8), socioeconomic systems (9 and 10).

Example 1. An electromechanical servosystem of rotation angle reproduction. Input g(t) is the adjustment of a new value for the rotation angle at the referenceinput element. Output y(t) is a rather smooth turn of the executive axis of control plant on a required angle. A control plant can be a physical object with large mass and moment of inertia: the large-sized directional antenna, telescope, launcher, rudder of a sea vessel, special purpose robotics handling actuator, etc. The final value of a rotation moment, explicated by an executive electric motor concerns the number of factors, defining the dynamic properties of a system, which restricts the possible angular acceleration of the executive axis with the defined reduced moment of inertia.

Example 2. A temperature stabilization system on a space orbital station bay. When the station passes a planet shadow, then the step decrease of the radiant energy stream from a star g(t) should be balanced by a connection between heatgenerating devices. However, their deduction of required thermal power is possible only under the smoother law y(t). Considering the stabilization of temperature in different parts of a bay also occurs only gradually.

Example 3. A system for the automatic tuning of radiation frequency in the jammer of radio interferences against missile control systems. At a step variation of carrier frequency of a suppressed transmitter signal g(t), the frequency of a generated radio interference varies under the more "smooth" law y(t) and can coincide with the signal frequency only a bit later. The value of this piece of time is the important characteristic of counter radio measurement efficiency. It is basically restricted from below due to the impossibility of the instant estimation of a signal frequency with the presence of noise, and also by limited operation speed of the radio interference generator device.

*Example 4. A digital control system of a plane when driving in a landing final approach.* Perturbation is a hard burst of wind in the direction, perpendicular to the runway axis, which leads to lateral displacement of the plane with values of discrete reference input g(nT), where  $T \approx 10^{-1}$ sec. The output is a deviation of the steering mechanisms which causes the lateral controlling force and gradual inverse lateral displacement of the plane with values of discrete reading y(nT).

*Example 5. A mans blood pressure stabilization system.* This action consists of the patient's reception of the defined dose of a medicinal preparation g(t). The re-

sponse is a smooth drop down of blood pressure y(t) with the onset of drug action and then in it's ending process — again a gradual rise of pressure. A correct choice of dosage and periodicity of medicine maximize the treatment efficiency.

*Example 6. A monitoring system for spreading infectious diseases.* This input action consists of the vaccination of a population in a given area, which ensures an increase of immunity to a disease agent, or other protective procedures. The result is the decrease of patients at the appropriate choice of action.

Example 7. A fish stores reproduction system in some fishing craft area. The action is a scheduled dropdown of annual catch g(nT), where T = 1 year, and the transfer of fish catching fleet from one area to another. A reaction is gradual increase of fish stores y(nT). The correct mathematical model of a system allows one to reasonably plan the results and to produce the additional fish guarding procedures in time.

*Example 8. A system for thermal balance maintenance of the Earth.* This anthropological action consists of sharp growth of mankind's energy needs, of which the significant part is produced at the expense of burning different kinds of chemical fuel. It leads, in particular, to the increase of carbonic gas percentage in the Earth's atmosphere g(t) and the "greenhouse effect" development that changes the balance of incidents on the Earth and emission by the Earth's radiant energy. This response consists in the increase of the annual temperature y(t) in characteristic geographical areas. The authentic mathematical model of this system has not yet been created.

Example 9. A governing system for branches of national industry (agriculture, transport, education, public health services). This input action is the additional tool or recourse allocated for development of manufacture in the current year g(nT), where T = 1 year. This response consists of increasing the amount or (and) production quality y(nT), worked out during the year. The possible criteria of assigned resource distribution are the minimization of time, for which the response rises up to a target level, and the maximization of the response level which is guaranteed to be achievable during the preset time, etc.

Example 10. Consumer demands prediction system for a given group of goods. The input is a variation of the goods price g(nT), where T = 1 day. The output y(nT) is the volume of sales estimation. Possible optimization criterion for function g(nT) consists of the maximization of the expected producer's profit.

Additional remarks to the dynamic system concept. The same real system depending on the research problem definition can be considered either as dynamic or as not dynamic. For example, the radar detector of a legitimate signal in the noise consisting of the matched filter and the threshold unit [34, 56] is a dynamic system, if the time lag between the occurrence of signal and threshold unit operation moment is investigated. If the detector operation speed is not being controlled, but only the probability performances of correct detection and false alert are of interest, then it is necessary to consider the detector as a non-dynamic system.

**Classification of dynamic systems.** The scheme of dynamic systems classified on character and on correlation of processes, taking place in them, is shown in Fig.1.2.

The linear dynamic system unlike the nonlinear one is described by a simple equation (differential or difference), as well as any part of a linear system, i.e. any dynamic unit, included in it. Dependence of a steady state output value from the fixed value of input  $x_2 = F(x_1)$  is called a static characteristic, for a linear unit or for a whole linear system this is represented by straight line (Fig.1.3*a*).



Fig. 1.2

The concept of a linear system actually is related to the accepted mathematical model, instead of real systems, any of which is certainly non-linear. However, with ranges of processes restricted by values, taking place in a system, the linear model in many cases represents quite adequately the properties, really shown by a system. First of all it is valid for the systems constituents of which have the restricted-linear static characteristics (Fig. 1.3*b*).

Remarkable, is that the impossibility to accept the linear model systems causes the essential complication of investigating its dynamic properties. Some of the most universal analytical methods of such investigation are coupled again to the substitution of nonlinear parts of a system by "equivalent" (in this or that sense) linear units. The methods of harmonic linearization, static linearization and many others follow this principle (see [9, 74, 80]).

Among the essentially nonlinear systems there are relay systems, in which the quantization of some process on a level takes place. Such systems include a unit or some units with a relay static characteristic (Fig.1.3 c, d).



Fig. 1.3

Independent of whether the system is linear or non-linear, it can be continuous or pulsed (sampled). The difference of pulsed systems from continuous ones is the available quantizations of any process or processes in time.

The pulsed and relay systems are joined together by one concept of discrete systems. There is a quantization of processes in time or (and) on a level in them.

A system, whose properties do not vary in time, is named stationary (really it is often possible to speak only about quasi-stationary). When the system proprieties vary essentially in time then it is named non-stationary.

In most cases the nonstationarity is coupled to unmonitored parametric or structural perturbations that cause the casual modifications of parameters or form of the equation describing a system, not having purposeful character.

In some highly organized dynamic systems the purposeful variation of dynamic properties for more successful system operation takes place under varying the ambient conditions.

If the variation of ambient conditions is applicable under the determined law, then the indicated positive effect is accessible in the program-tunable systems, whose properties vary under the defined fixed program. An example is the seasonal reorganization in photochemical synthesis systems for over-year plants.

If the law of ambient conditions variation beforehand is unknown, then the influence of these modifications on relative successfulness of system operation can be removed in adaptive systems, where the reorganization is carried out depending on continuously analyzable ambient conditions. An example is a radar servomechanism with adaptation depending on a power or other properties of radio interferences [34].

An adaptive system with tunable parameters is called a self-adapting system and the system with tunable structure — a self-organizing one.

The stationary system depending on those or other dynamic properties can be referred either to stable, or unstable systems (the intermediate state corresponds to the stability limit). These concepts will be illustrated further in §1.2. Remarkable is that the defined fixed status of non-stationary system can also be stable or unstable. If the stable system status (unstable status theoretically can also be taken as a basis, however it would not be practical) is kept in the presence of small parametric perturbations then such a system is named rough, otherwise — not rough.

#### §1.2. Stable and rough systems

Stability. The concept of stability is applicable to any dynamic system.

The system is called stable, if it recovers back to an initial state after removal of perturbation. Perturbation plays the role of any external action. Thus the initial undisturbed state can be implied with not only static status with constant inputs and outputs, but also the status described by variation of input and output values in the interacted laws.

The stability of linear system is equal to the property exclusive of the system that does not depends on excitations, applied to a system or having been applied to it earlier. A convenient test input action g(t) when estimating a linear system stability is the short rectangular pulse with a final square (in a limit —  $\delta$ -function).

If the response of a system on such input y(t) at  $t \to \infty$  tends to zero, the system is stable. If the response is restricted on magnitude, the system is on the limit of stability. If the response is not restricted, the system is unstable. It is illustrated by the graphs in Fig. 1.4 *a*, *b*, and *c* accordingly.



The stability of nonlinear system can depend on external actions and is defined in a special way with the account of their value.

The analytical stability conditions of any dynamic system can be formulated only after the representation of its mathematical model is provided. These conditions look like inequalities written for parameters of a system.

**Stability regions**. Geometrical maps of stability conditions contain the stability regions constructed in a plane (generally — in multidimensional space) of system parameters.

The possible kind of stability regions when analyzing two parameters A and B is shown in Fig.1.5.



Fig. 1.5

**Concept of system roughness**. If the stability region has significant or even indefinitely large width in all directions (Fig.1.5*a*, *b*), then the system can have roughness property or else is rough. In a rough system with nominal values of parameters, lying far enough away from stability limit, then the stability is kept at possible deviations of parameter's values away from nominal ones.

If the stability area has a small (in a limit — infinitesimal) width in some or in all directions (Fig. 1.5c, d), the system is not rough.

If the stability area is completely absent from a plane (generally — in multidimensional space) of all system parameters, then the system is called structurally unstable.

Naturally, in a society and in engineering both unstable and non-rough dynamic systems, as a rule, are impractical, nonviable and have no working capacity.

## §1.3. Influence of external action properties on indexes of system successful functioning

**Multidimensional and multicoupled systems.** In real complicated dynamic systems there are usually some inputs (input actions) and some outputs (responses). For each possible pair of input-output it is possible to define some dynamic properties, which completely characterize such a system, called multidimensional. For example, the linear multidimensional dynamic system is completely characterized by a matrix transfer function *H*, linking a matrix-column of action representations  $G = [g_1g_2...g_m]^T$  and matrix-column of responses representations  $Y = [y_1y_2...y_m]^T$  by the relation Y = HG.



Fig. 1.6

In special cases the multidimensional system can have one input and some outputs (then H is a functional matrix-column) or some inputs and one output (then H is a functional row matrix).

If each of the outputs in a multi-dimensional system depends only on one of the inputs (Fig.1.6*a*), then the system can be represented as several off-line channels with no cross-feed (the functional rectangular matrix H is diagonal in this case). Otherwise (Fig.1.6*b*) the system is multicoupled.

**Types of external actions**. In the specific problems of systems research one input and one output are often discriminated, and the systems are multicoupled, for example,  $g = g_1$  and  $y = y_1$  in Fig.1.6b. The interaction between input and an output represents the main interest in the given concrete problem. Thus the dependence of response on other input actions only spoils the determination of the main investigated dependence. In this case it is necessary to consider such actions in a concrete considered problem such as noise or disturbance.

In a linear multicoupled system, where the principle of superposition is fulfilled, the reaction from a disturbance can be immediately added to a reaction from the main action. If the dynamic properties of transmission channels in the main action and the disturbances coincide then it is possible to consider, that the action and the disturbance are applied to one input or are reduced in one input as a result of equivalent transformations of the system model (Fig.1*b*, *c*). It allows for the consideration of more simple research of the problem of a single-channel system with an additive mixture of the fundamental action and interference on input instead of the research of the problem of a multicoupled system.

The reaction of interference in a nonlinear multicoupled system can lie in a rescaling of an output reaction from the fundamental action. In this case the interference is called multiplicative. The same interference can have both additive and multiplicative components in practice. It is possible to allocate the other characteristic case, when the interference action causes the value of some parameter to vary in the mathematical model of the dynamic system. Such interference is called parametric or parametric perturbation. There is a concept of structural perturbation leading to a variation of the kind mathematical formulas, describing a system, i.e. to a modification of system structure.

The example of parametric perturbation in a plane control system can be a variation of rudder efficiency owing to a variation of the planes' aerial velocity or air density, the example of structural variation or the jamming of rudder.

**Factors of successful system operation.** Stability and roughness properties still do not ensure the successful operation of dynamic system or, as it is usually spoken of in an engineering system, it's working capacity. These properties create only the necessary background for this purpose. The system operation is considered successful only when the values of major numerical factors describing it lye in the permissible limits. The indicated factors are included in some formal successful condition of operation permitting selection of the subset of successfully operating variants of dynamic systems from a set of actual or hypothetical ones.

The choice of particular quantitative factors for successful system operation depends on many factors and is not always obvious. If the system is natural and is not created by men, then the fact of such system existence usually testifies the system efficiency and acceptable values of all quantitative factors. However, someone can evaluate even such a dynamic system by the subjective criteria. For example, people have individual conceptions about an ideal climate or weather characterized by air temperature, wind, rainfall amount and other parameters. Someone concludes a measure of atmospheric dynamic processes behavior depending on actual values deviations of such parameters from the "ideal". But the successful operation conditions of engineering dynamic systems must be formulated with considerably less voluntarism.

Consider, that some factor of successful operation I is given in one way or another, and it is evaluated depending on two factors:

• Dynamic properties of a system described by some operator or a set of operators *H*;

• Regularities of external actions variation described by some set of characteristics *S*.

The example of an operator H in the case of a linear system can be its transfer function. Examples of characteristics S are the spectral densities, and at the determined approach particular analytical expressions or graphs of external actions.

The factor of successful functioning generally can be a vector in *m*-dimensional space  $I = (I_1, I_2, ..., I_m)$ . Its components are the scalar values  $\{I_i\}_1^m$ , comprehensively describing, for example, the possible deviations of system response from the values, which would correspond to its "ideal" (in the sense of the adopted criterion) operation at different external actions. In the case of servo-mechanism the components of vector *I* can be, for example, its maximal error, root-mean-square error, constant component of error, decay time of step response, maximal relative value of system overshoot in such step response and other similar characteristics.

Knowledge of factor I = I (*H*, *S*) allows finding out in each particular case, whether the operation of dynamic system is successful. For this purpose the region of satisfactory values for this index should be allocated in *m*-dimensional space which will be denoted as  $I_{sat}$ .

The operation of a system with the defined dynamic properties H at external actions with characteristics S is successful, when it satisfies the condition

$$I(H,S) \subset I_{sat} \,. \tag{1.1}$$

Otherwise the system operates poorly or, when speaking with reference to engineering systems, has no working capacity.

Underline that the condition (1.1) is written at quite defined and univocally known characteristics H and S.

#### §1.4. Robust systems

**Robustness concept.** Evaluating the successfulness of system operation on the base of (1.1), it is necessary to take into account, that due to the parametric and structural perturbations the dynamic properties of system can vary or have a scatter for different copies of a system. Only the class of possible dynamic properties of system  $M_H$  is supposedly known, i.e. for each particular system the accurate kind of operator H is unknown, but the following condition is certainly satisfied

$$H \subset M_H. \tag{1.2}$$

For conditions of system operation and appropriate characteristics of external actions S the complete determinacy is not usually proper. A particular kind of these characteristics is considered unknown, but some of their class  $M_S$  is given, and the following condition is necessarily satisfied

$$S \subset M_S. \tag{1.3}$$

Now evaluate the realization of condition (1.1) at all admissible characteristics H and S, corresponding to conditions (1.2) and (1.3), which can then be mathematically written as:

$$I(H, S) \subset I_{sat}.$$

$$\underset{\substack{H \subset M_H \\ S \subset M_S}}{(1.4)}$$

If condition (1.4) is fulfilled, then the system has a property of robustness and is called robust. If condition (1.4) is not fulfilled, the system is not robust, but still can be stable and rough.

The robust property means saving the beforehand assigned system operation performance at a high enough level. It must be fulfilled at all possible characteristics of external actions within the framework of some of their preset class at the account of an admissible variation of system dynamic properties due to parametric and structural perturbations.

For simplification of condition (1.4) entry, general input actions, disturbances and also parametric and structural perturbations will be referred further to external actions (its properties are described by a set of characteristics S). Thus it will be considered that the operator H characterizes the nominal dynamic system properties, defined at absence of parametric and structural perturbations. Then condition (1.4) looks like

$$I(H,S) \subset I_{sat}, \tag{1.5}$$

The most unfavorable properties of actions. Suppose some formal rule is chosen, permitting us to define the kind of different successfully enough operating variants of dynamic system, according to the condition (1.5), operating more successful than others. Actually it means that the vectorial successfulness factor of operation  $I = (I_1, I_2, ..., I_m)$  is replaced by scalar or is converted into a scalar factor. For example, it can look like

$$I = \sum_{i=1}^{m} \alpha_i I_i , \qquad (1.6)$$

where  $\{\alpha_i\}_1^m \in [0,\infty)$  are weight coefficients. In that specific case all weight coefficients in (1.6), except only one, can be equal to zero.

In that particular case it will be considered, that the smaller value of index I corresponds to the more successful system operation. Still factor I is a function of nominal system dynamic properties H and properties of external actions S (i.e. properties of the fundamental input action, disturbance, parametric and structural perturbations).

It is possible to find such properties of actions  $S = S_{mu}$  within the framework of their preset class  $M \subset M_S$ , at which the system with the defined dynamic properties *H* will operate least successfully and the value of the factor *I* will reach maximum

$$I(H, S_{mu}) = \max_{S \in M_S} I(H, S).$$

The actions with the indicated properties  $S_{mu}$  are considered the most unfavorable.

It is possible to find the optimal dynamic properties of a system  $H_{opt}$ , which provides the most successful operation at the defined properties of external actions *S*, i.e. the minimal value of the factor

$$I(H_{opt}, S) = \min_{H} I(H, S).$$
(1.7)

**Minimax robust systems.** Set the problem of dynamic system properties optimization by criterion (1.7), under the requirement that the external actions necessarily have the most unfavorable properties  $S = S_{mu}$  at any selected operator of system *H*. Such a task can be treated from the positions of game theory with a two player game. One of the players gives orders for choice of external actions properties and tends to maximize the factor *I*, and another one gives orders for choice of system properties with the purpose to minimize this factor. As a result of solution the minimal guaranteed value of successfulness index of system operation can be found as: